Quiz 3

MTH 403, Semester 1, 2021-22

Duration: 55 minutes

Maximum Points: 30

Directions

- *While writing solutions, please ensure that you sufficiently explain and justify all intermediate arguments leading to any conclusions that you may draw along the way.*
- *Each statement (or argument) in your solution must be clearly explained, and must be devoid of any logical fallacies or gaps.*
- *The third question is a bonus question which carries an additional 10 points. You are advised to attempt this question only after you have tried solving all remaining questions.*

Questions

1. Consider $\alpha = \dfrac{1}{2\pi}\dfrac{x\,dy - y\,dx}{x^2+y^2} \in \Lambda^1(\mathbb{R}^2 - \{0\})$. [5+10]
   (a) Show that α is closed.
   (b) Show that α is not exact. [Hint: Integrate α on the unit circle.]
2. For $\theta \in [0,1]$, let $\lambda_{r,n}(\theta) = (r\cos(2\pi n\theta), r\sin(2\pi n\theta))$. [10+5]
   (a) Show that there exists a unique number n such that $\int_{\lambda_{r,n}} d\theta = 2\pi n$.
   (b) Show that there exists no 2-chain c in $\mathbb{R}^2 - \{0\}$ such that $\lambda_{r,n} = \partial c$.
3. **(Bonus)** Use 2(b) to establish the fundamental theorem of algebra. [10]